AL^{+} : On the Extension of Argumentation Logic (AL)

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Background

- Classical Deduction (PL) can be reformulated in Argumentation Logic (AL)
  - AL≡PL
  - AL does not explode.
- Can we extend AL to $AL^{+} \supset AL$?

Technical Background (Informal)

- **AL Reasoning carried out via Dialectic Argumentation between a formula φ and all its opposites.**

- **AL Reasoning for φ via Cases**
  - **A case for φ**
  - **No case for ¬φ**

- **AL Reasoning via "Case Satisfiability".**

What is a Case?

- A Case is a set of arguments - formulae Δ that deals with/defends against all opposing sets A.

- Defending against A via:
  - Take in the Case a directly opposite position
    - Include in Case ¬ψ for some ψ in A - Undermine A
  - A is opposed by the given premises T.

Formal Definition of Argumentation Logic

In ABSTRACT Argumentation

<Args, ATT>/<Args,Att,Def>

- ***Acc(Δ,Δ')***: Set *Δ* is **acceptable relative** to a set *Δ'*
- *A* ***relative*** ***Case*** *for Δ in the context of Δ'*

Relative Acceptability Semantics
<Args,Att,Def>

***Acc(Δ,Δ')*:** Set Δ is **acceptable relative** to a set Δ'

- ***Acc(Δ,Δ')*** iff $\Delta \subseteq \Delta'$, or
  for any **A** that **attacks** Δ: $A \not\subseteq \Delta' \cup \Delta$
  there exists **D** that **defends against A**
  such that ***Acc(D, Δ' ∪ Δ).***
- ***Acc(-,-)*** is the **least fixed point of the *Acc operator***.

Δ is **acceptable** iff **Acc(Δ,{})** holds

Computation of Relativistic Argumentation <Args,Att,Def>

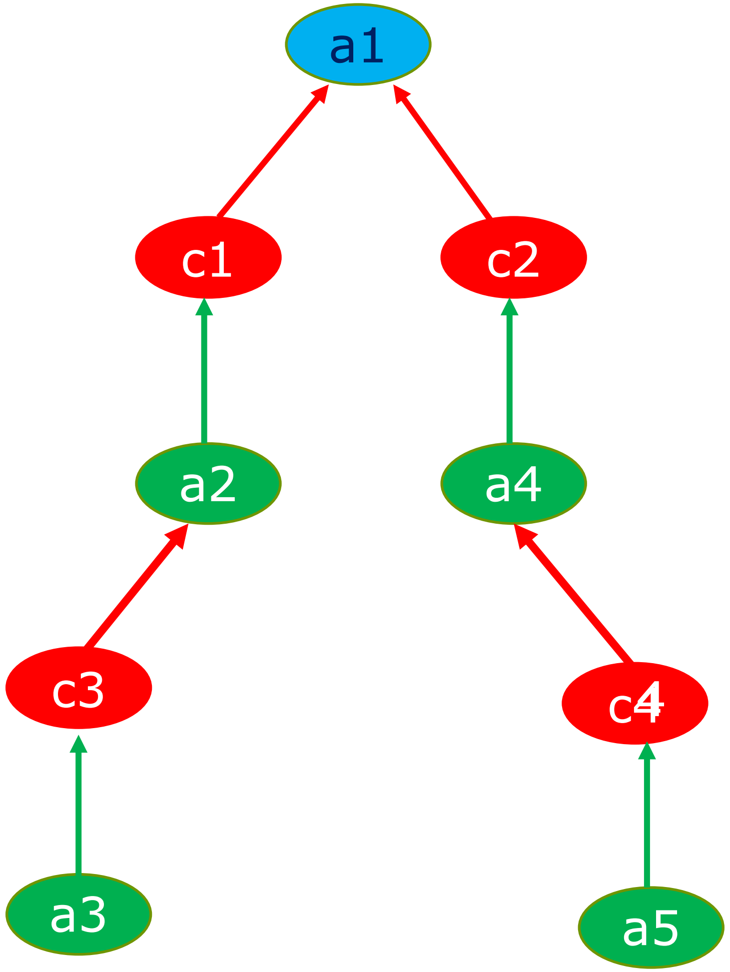
Terminating condition for acceptability:

- A **defence** belongs to earlier **defences**.

a3= a2 (or a1)

Terminating condition for Non-acceptability:

- An **attack** belongs to earlier **defence**.



In general, it is more complicated, e.g., may need to consider non-minimal attacks:

[F. Toni Thesis & …]

Argumentation Logic ≈ Propositional Logic

- **Relative Acceptability in <Args, Att, Def> for PL where**

  - **Args: Identified with Sets of Propositional Formulae: Δ**

    Support formulae via direct schema proofs from Δ

  - **Att: A attacks Δ : $T \cup \Delta \cup A \vdash_{DD} \bot$**

  - **Def: (i) "¬φ defends against φ and vice-versa, i.e. Freedom of Choice"**

    (ii) Arguments from the given theory T defend against those outside T but not vice-versa.

Argumentation Logic ≈ Propositional Logic

Deduction via Relativistic Argumentation

T = {}

Example of Excluded Middle Law: $q \vee \neg q$

{¬(q v ¬ q)}

↑ attacks

{q}

⇑ defends

{¬q}

↑ attacks

{¬(q v ¬ q }

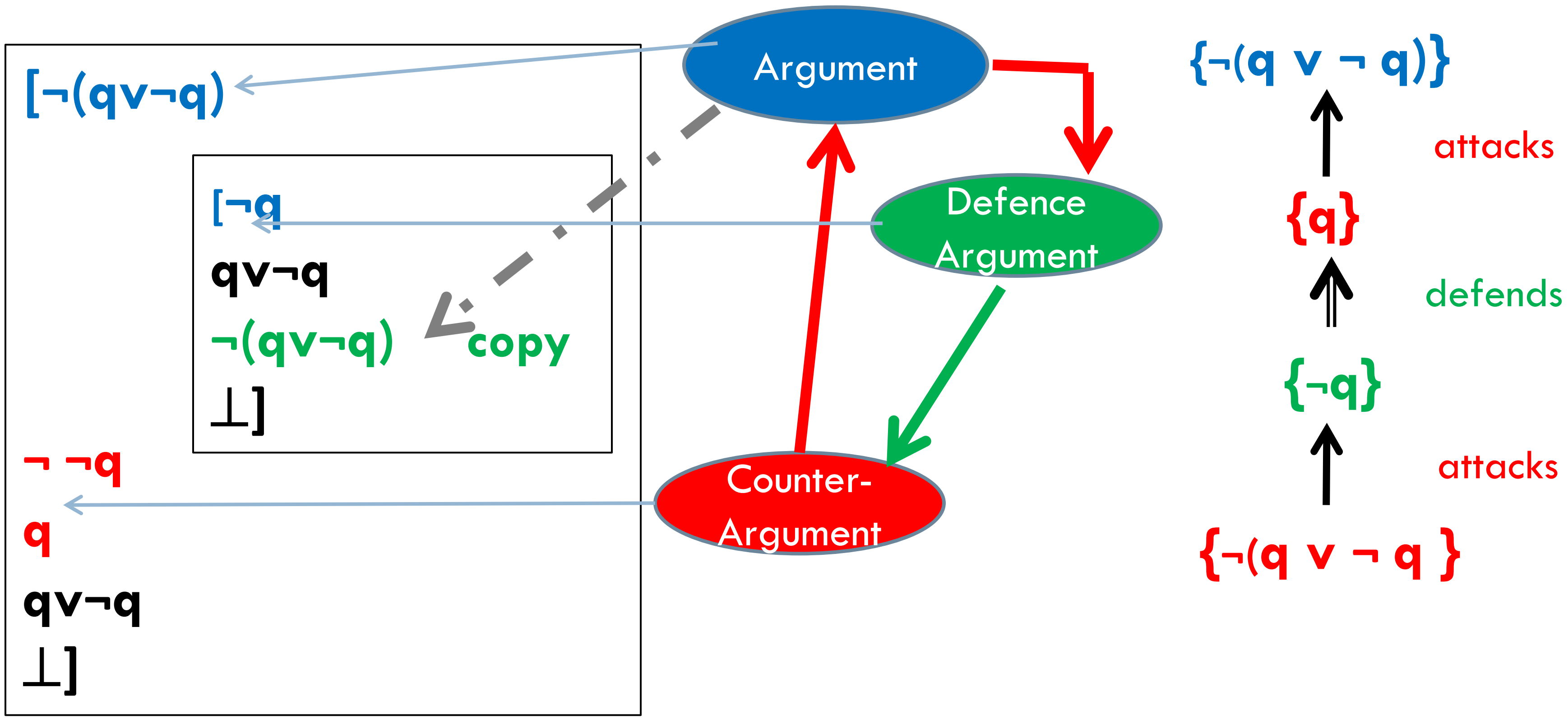
Hence ¬(q v ¬ q) is non-acceptable.

AL = PL rests on ¬ Acc(φ) ⇔ φ is inconsistent (via RAA)

Reduction ad Absurdum (RAA) in AL

AL = PL rests on ¬ Acc(φ) ⇔ φ inconsistent (via RAA)

T2 = {} - Excluded Middle Law



¬ Acc(ϕ) ⇔ ϕ inconsistent (via RAA)

Reduction ad Absurdum (RAA) in AL

RAA ⇔ Non-acceptability

But

$\neg$ Acc(φ) $\not\Rightarrow$ Acc($\neg$φ)

Inconsistent φ $\not\Rightarrow$ Entailment of $\neg$φ

$\not\Rightarrow$ Consistency of $\neg$φ

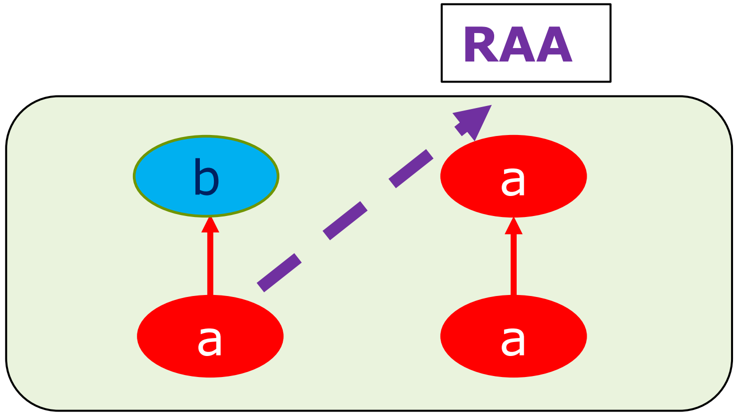
Both $\neg$ Acc(φ) and $\neg$ Acc($\neg$φ) can hold (as in logical paradoxes)

AL^+ - Extending AL

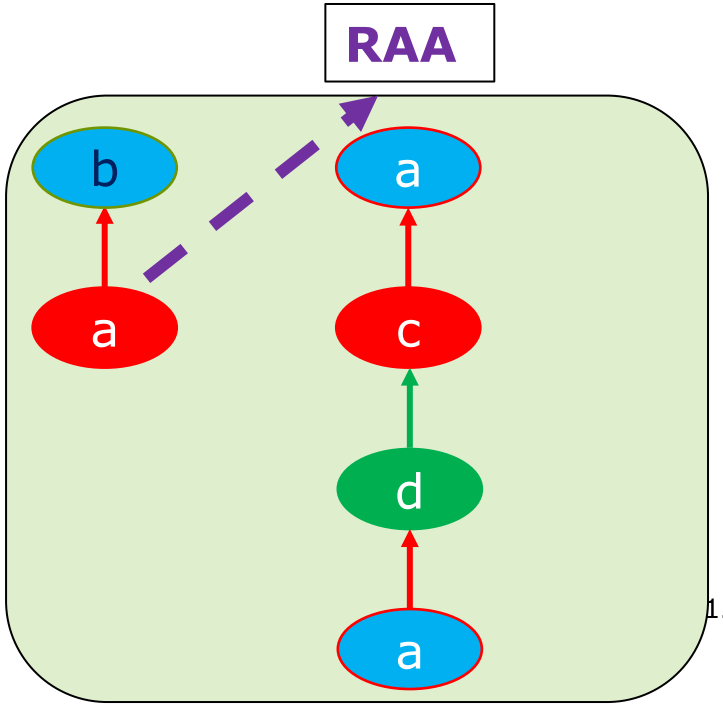
Examples in Abstract Argumentation

- **Historical Background**

- **Pisa 1991: LP with Inconsistent NAF Literals/Assumptions**
  - **Beyond Admissible Negation as Failure**
  - **Acceptability Semantics for NAF**



$Acc^+(b,\{\})$



AL^+ from AL - Formal Definition

$Acc^+(\Delta, \Delta_0)$ iff $\forall$A attacks Δ (A $\not\subseteq$ Δ U Δ_0):

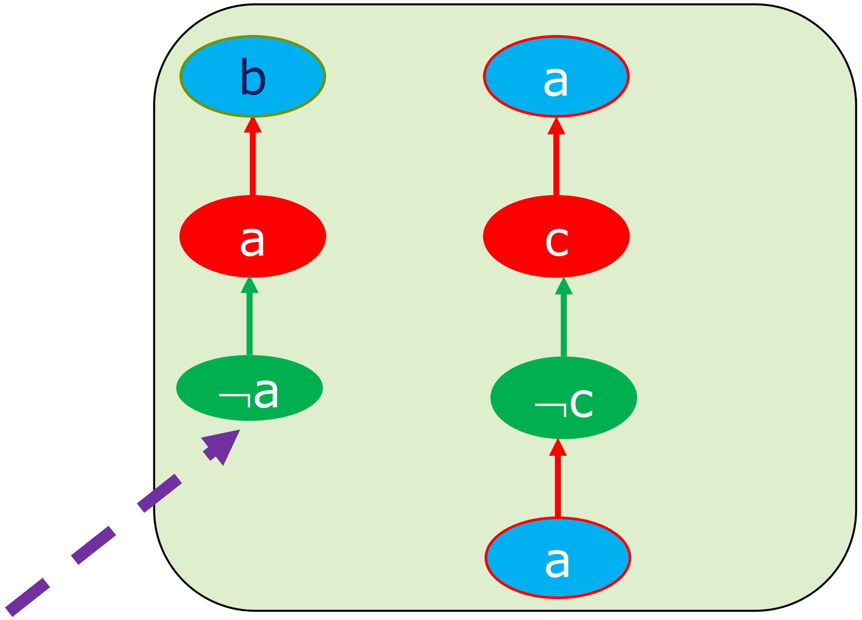
- $\exists$ Δ' s.t. defends against A and $Acc(\Delta', \Delta \text{ U } \Delta_0)$

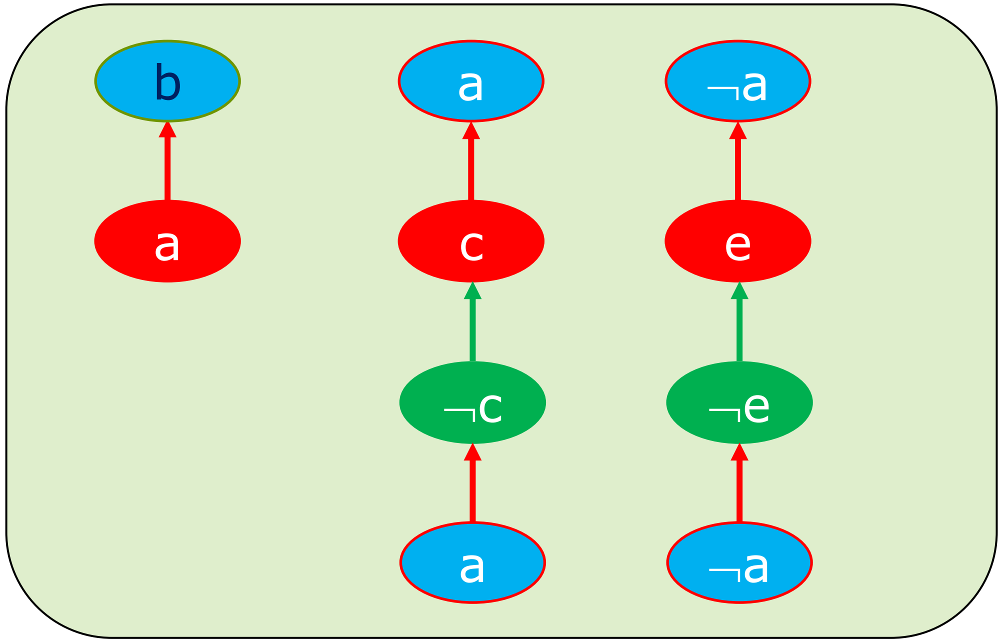
OR

- $\neg$ Acc(A,{})

- **Results:**
  - AL^+ does not explode AL in PL (for consistent T)
  - $AL^+ \supset$ AL

$AL^{+} \supset AL$ – Example 2 cnt.





Case of: **LOGICAL PARADOX**

Argumentation Logic
Beyond Classical Logic

- **Argumentation Logic applies unchanged when premises in T are inconsistent**
  - **No explosion or trivialization.**
  - **Inconsistency/Paradoxes => Alternatives**
- **Can extend Defense/Preferences:**
  - **Direct Conflict subsets of T defend each other**
  - **Domain Preferences on T or arguments**

Closing the Circle of AL and Argumentation in AI, e.g. of NM Logics, etc in AI

Wider Scope:
What is Logical Reasoning?

Consistency & Entailment

vs

Freeness & Satisfiability

Verify Consistency

vs

Build Acceptable Cases

(Aristotle: Non-self-contradictory Argument)